LOYAL EDUCATION
Result Oriented

M A T H E M A T I C S

(NEET & JEE MAINS)

Types of Polynomials

Based on highest degree

* **Linear Polynomial** : Any polynomial which is in the form of $ax + b$, where $a \neq 0$ and a, b being real numbers is called a linear polynomial.

  Example: $2x+3$, $5x + 4$, $-5x - 3$

  Note :
* 'ax + b' is also called the standard form or general form of a linear polynomial.
* Every linear polynomial has two terms.
* The first term 'ax' is called the x-term.
* The second term b is called the x^0 term (or) constant term (or) independent term.
* The highest degree of every linear polynomial is '1' (one).
* Every linear polynomial has only one solution called factor (itself).
* **Quadratic Polynomial** : Any polynomial which is in the form of $ax^2 + bx + c$ where $a \neq 0$ and a, b, c being real numbers is called a quadratic polynomial.

  Example : $x^2 + 3x + 2$, $2x^2 - 5x - 7$, $x^2 - 5x + 6$ etc.

  Note :
* $ax^2 + bx + c$ is called the standard form (general form) of a quadratic polynomial.
* Every quadratic polynomial has three terms in it.
* The first term ax^2 is called 'x^2 term.
* The second term bx is called 'x' term.
* The third term c is called x^0 term or constant term or independent term.
* Every quadratic polynomial has exactly two solutions called factors
* If the two zeros of a quadratic polynomial f(x) are denoted by α, β, then the formula to find it is given by

  $f(x) = k\,[x^2 - (\text{sum of the zeros})\,x + (\text{product of the zeros})]$; i.e.

  $f(x) = k\,[x^2 - (\alpha + \beta)\,x + \alpha\beta\,]$; where the relationship between the zeros and the co-efficients is given by

  $$\alpha + \beta = -\frac{\text{co}-\text{efficien of x}}{\text{co}-\text{efficient of } x^2} = -\frac{b}{a} \quad \text{and} \quad \alpha\beta = \frac{\text{constant term}}{\text{co}-\text{efficient of } x^2} = +\frac{c}{a}$$

* <u>Cubic polynomial</u>: Any polynomial which is in the form of $ax^3 + bx^2 + cx + d$, where $a \neq 0$ and a, b, c, d being real numbers is called a cubic polynomial.

  Ex: $x^3 - 3x^2 - 3x + 1$, $x^3 + 5x^2 - 2x + 10$

  <u>Note :</u>
* $ax^3 + bx^2 + cx + d$ is also called the standard form of a cubic polynomial.
* Every cubic polynomial consists of 4 terms.
* The first term ax^3 is called x^3 term.
* The second term bx^2 is called x^2 term.
* The third term cx is called x term.
* The fourth term d is called x^0 term or constant term or independent term.
* Every cubic polynomial has exactly 3 solutions called factors.
* If the three zeros of a cubic polynomial f(x) are denoted by α, β, γ, then the formula to find it is given by

  f(x)= k [x^3- (sum of the zeros)x^2 + (sum of product of zeros taken two at a time)x - (product of the zeros)]; i.e.

  $f(x) = k[x^3 - (\alpha + \beta + \gamma)x^2 + (\alpha\beta + \beta\gamma + \gamma\alpha)x - \alpha\beta\gamma]$; where the relationship between the zeros and the co-efficients is given by:

$$\alpha + \beta + \gamma = -\frac{\text{co-efficient of } x^2}{\text{co-efficient of } x^3} = -\frac{b}{a} \quad \text{and} \quad \alpha\beta\gamma = -\frac{\text{constant term}}{\text{co-efficient of } x^3} = -\frac{d}{a}$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = -\frac{\text{co-efficient of } x}{\text{co-efficien of } x^3} = \frac{c}{a}$$

* <u>Biquadratic Polynomial</u>: Any polynomial which is in the form of $ax^4 + hx^3 + cx^2 + dx + e$ where a, b, c, d, e, being Real numbers, $a \neq 0$ is called a biquadratic polynomial.

  Example: $6x^4 + 3x^3 - 2x^2 + 6x + 1$

  <u>Note</u>:

- The standard form of a biquadratic polynomial is $ax^4 + hx^3 + cx^2 + dx + e$.
- Every biquadratic polynomial has 5 terms.
- The first term ax^4 is called 'x^4' term.
- The second term bx^3 is called 'x^3' term.
- The third term cx^2 is called 'x^2' term.
- The fourth term dx is called 'x' term.

- The fifth term e is called 'x^0' term or constant or independent term.
- Every biquadratic polynomial has exactly 4 solutions called factors.

Types of Polynomials

Based on number of terms

- Monomial: Any polynomial which has 1 term is called a monomial.
  Ex: x^2, 2xy, -x, xyz.......etc.
- Bionomial: Any polynomial which has 2 terms is called a Bionomial
  Ex: $x + 2$, $3x^2 + 5$, $4x^2 + 5$, $4x^3 + 2x^2$ ....etc.
- Trinomial: Any polynomial which has 3 terms is called a trinomial
  Ex: $x^2 + 3x + 2$, $x + y + z$, ... etc.
- Zero of a polynomial: If f(x) be any given polynomial and 'a' being a real number and if f(a)=0 then a is called the zero of the polynomial f(x).
- Factorisation: A procedure (method) in which a given polynomial is expressed as the product of its maximum number of factors is known as factorisation.
- Solve : Finding all the possible values of the unknown quantity or quantities, which exists in the problem is called solving.
- Remainder Theorem: If f(x) be any given polynomial of degree greater than or equal to 1 and small 'a' be any real number, such that f(x) is divided by (x - a), then the remainder is equal to f(x).
  $f(x) = (x - a)\, q(x) + r(x)$

Note:

* f(x) is called the dividend
* (x - a) is called the divisor
* q(x) is called the quotient
* r(x) is called the remainder.
* If a polynomial p(x) is divided by (x + a) the remainder is the value of p(x) at x = -a i.e., p(-a).
  $[x + a = 0 \Rightarrow x = -a]$.
* If a polynomial p(x) is divided by ax - b the remainder is the value of p(x) at x = -a i.e., p(-a).

[x + a = 0 => x = -a]

* If a polynomial p(x) is divided by ax + b the remainder is the value of p(x) at x = -b/a i.e., p(-b/a).
* If a polynomial p(x) is divided by (b - ax) the remainder is the value of p(x) at x = b/a i.e., p(-b/a).
  [b - ax = 0 => -ax = -b => x = -b/-a => x = b/a]
* Remainder theorem is used to find the remainder.
* Remainder theorem is also called the division rule i.e.
  Dividend = Division x Quotient + Remainder.
* <u>Factor Theorem</u> : If f(x) be any given polynomial of degree greater than or equal to 1 and small 'a' be any real number, such that f(a) = 0, then (x - a) is a factor of f(x).

<u>Note:</u>

* (x + a) is a factor of a polynomial f(x) if f(-a) = 0
* (ax - b) is a factor of a polynomial f(x) if f(b/a) = 0
* (ax + b) is a factor of a polynomial f(x) if f(-b/a) = 0.
* (x - a) is a factor of a polynomial f(x) if f(a) = 0
* Factor theorem is used to factorize a polynomial by division method or Horner method.